تحلیل گر: دکتر آرش دیو سعی

۱) (متوسط)

هندسی: $a \,,\, ar \,,\, ar^2$

نصف کن

حسابی: $\frac{a}{2} \,,\, \frac{ar}{2} \,,\, \frac{ar^2}{2}$

$$ar = \frac{a}{2} + \frac{ar^2}{2} \xrightarrow{\times 2} 2r = 1 + r^2 \longrightarrow (r-1)^2 = 0$$

$r = 1 \rightsquigarrow d = 0$

(گزینه ۲)

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۲) (مفهومی متوسط و وقت‌گیر)

$$y = a(x+1)^2 + 1$$

$$\Rightarrow y = ax^2 + 2ax + a + 1$$

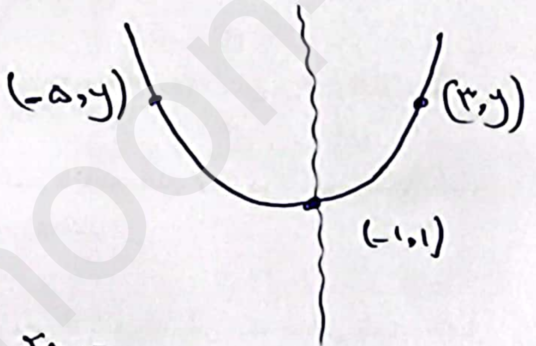
$\alpha + \beta = -2 = S$

$\alpha \cdot \beta = \frac{a+1}{a} = P$

$\alpha^2 + \beta^2 = 5 \Rightarrow S^2 - 2P = 5$

$4 - \frac{2a+2}{a} = 5$

$a = -\frac{2}{3} \longrightarrow y = -\frac{2}{3} + 1 = \frac{1}{3}$



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۳) (متوسط و وقت‌گیر)

$$\frac{1}{x^2} + \frac{1}{(1-x)^2} = \frac{140}{9} \Rightarrow \frac{(x-1)^2 + x^2}{x^2(x-1)^2} = \frac{140}{9}$$

فرض: $x^2 - x = t \Rightarrow \frac{2t+1}{t^2} = \frac{140}{9} \Rightarrow 140t^2 - 18t - 9 = 0$

$140t^2 - 4t - 1 = 0 \Rightarrow t^2 - 4t - 140 = 0 \Rightarrow (t-8)(t+5) = 0$

روش ac

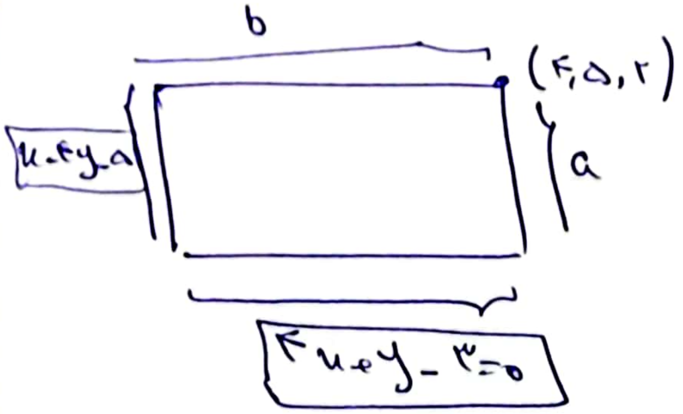
$t = \frac{8}{140}$ ، $t = -\frac{5}{140}$

$t = \frac{3}{10}$ ، $t = -\frac{3}{14}$

$x^2 - x - \frac{3}{10} = 0 \longrightarrow S = 1$

$x^2 - x + \frac{3}{14} = 0 \longrightarrow S = 1$

(۲)



خوب بود

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$$\begin{cases} x - 4y = 5 \rightarrow \boxed{m = 1/4} \\ 4x + y = 3 \rightarrow \boxed{m = -4} \end{cases}$$

→ دقیق ریشه ها

$$a = \frac{17}{\sqrt{17}} \div 2 = \boxed{\frac{17}{2\sqrt{17}}}$$

$$b = \frac{8.5}{\sqrt{17}}$$

۸

$$f(x) = \sqrt{x - 2\sqrt{mx - 1}}$$

$$y = 12 - x$$

$$f(m+4) = ?$$

خوب بود

$$y^{-1} = 12 - x$$

$$f^{-1}(2) = 10$$

$$10 = 12 - x \Rightarrow \boxed{x = 2}$$

$$f(10) = 2 \longrightarrow 2 = \sqrt{10 - 2\sqrt{10m - 1}} \Rightarrow 10m = 10 \Rightarrow \boxed{m = 1}$$

$$f(m+4) \xrightarrow{m=1} \boxed{f(5) = 1}$$

۹

$$n_2 = n_1 \left(\frac{8}{9}\right)^{t/2} \Rightarrow \frac{1}{4} = \left(\frac{8}{9}\right)^{t}$$

$$n_2 = \frac{1}{4} n_1$$

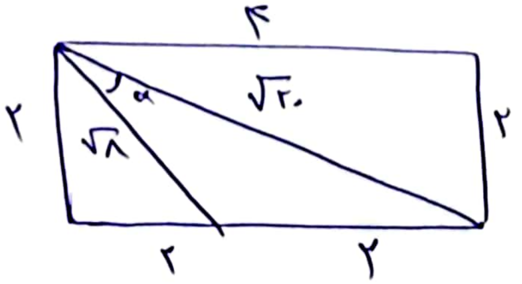
$$4 = \left(\frac{9}{8}\right)^{t} \Rightarrow \boxed{\log_5 4 = t\left(\log_5 9 - \log_5 8\right)}$$

$$\begin{cases} \log_5 2 = \frac{5}{12} \\ \log_5 3 = \frac{5}{7} \end{cases} \Rightarrow \frac{5}{12} + \frac{5}{7} = t\left(\frac{10}{7} - \frac{15}{12}\right)$$

$$\Rightarrow t = \frac{19}{3} \times 60_{min} = \boxed{380 \text{ min}}$$

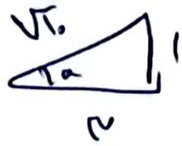
آسان تمام

(۱۰) (متوسط)



$$S = 8 - 2 - 4 = 2 = \frac{1}{2}\, 2\sqrt{2} \times 2\sqrt{5}\, \sin\alpha$$

$$\sin\alpha = \frac{1}{\sqrt{10}} \Rightarrow$$



$$\Rightarrow \boxed{\cot\alpha = 3}$$

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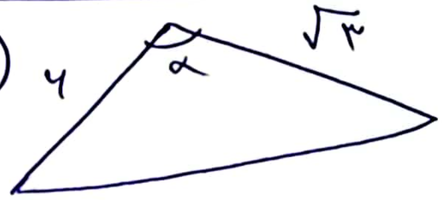
(۱۱) (آسان)



$$S = 4.5 = \frac{9}{2} = \frac{1}{2}(6)\sqrt{3}\,\sin\alpha$$

$$\sin\alpha = \frac{\sqrt{3}}{2} \Rightarrow \boxed{\alpha = 60, 120}$$

✓ ✓

۲ جواب قابل قبول هستند

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(۱۲) (متوسط و خوب)

$$f(x) = a + \frac{b}{2}\sin\left(2cx - \frac{3\pi}{2}\right)$$

$$\underbrace{\sin\left(2cx - \frac{3\pi}{2}\right)}_{\cos(2cx)}$$

$$f(x) = a + \frac{b}{2}\cos(2cx)$$

$$f(x) = 1 - 2\cos(2cx)$$

$$T = \pi = \frac{2\pi}{2c} \Rightarrow \boxed{c = 1}$$

$$\cos(2x) = \frac{1}{2} \to \begin{cases} 2x = \frac{\pi}{3} \\ 2x = 2\pi - \frac{\pi}{3} \end{cases}$$

$$\begin{cases} x = \frac{\pi}{6} \\ x = \frac{5\pi}{6} \end{cases} \Rightarrow \xrightarrow{\text{اختلاف}} \frac{2\pi}{3}$$

$\cos x - \sin x = t$ (۱۳) متوسط

$t^2 = 1 - \sin 2x$

$mt - 2\sqrt{4}(1-t^2) = \sqrt{4}$

$\Rightarrow \boxed{2\sqrt{4}t^2 + mt - 6\sqrt{4} = 0} \Rightarrow 2\sqrt{4} + m\sqrt{\frac{2}{3}} - 6\sqrt{4} = 0$

$\Rightarrow \boxed{m = 4}$

$\cos(x + \pi/4) = \frac{1}{\sqrt{3}}$

$\cos x \cos \pi/4 - \sin x \sin \pi/4 = \frac{1}{\sqrt{3}} \Rightarrow \cos x - \sin x = \sqrt{\frac{2}{3}} = t$

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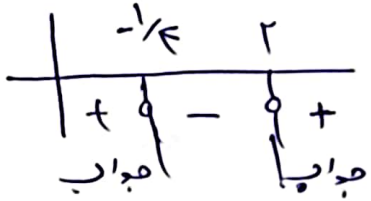
(۱۴) متوسط

f نزولی $\longrightarrow f(m^2 - m - 5) < f(-3 + 2m - m^2)$

$m^2 - m - 5 > -3 + 2m - m^2$

$2m^2 - 3m - 2 > 0 \Rightarrow$



$m = 1$ یک مقدار صحیح

قابل قبول نیست

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(۱۵)

$f(x) = \frac{ax+b}{cx+d}$

$g(x) = \frac{cx+d}{ax+b}$

$g^{-1}(x) = \frac{-bx+d}{ax-c}$

$\lim_{x \to -\infty} \frac{f(x)}{g^{-1}(x)} = \lim_{x \to +\infty} \frac{g^{-1}(x)}{g(x)}$

$\frac{\frac{a}{c}}{-b/a} = \frac{\frac{-b}{a}}{\frac{c}{a}}$

$a^2 = b^2$

$\Rightarrow a = \pm b$

$\lim_{x \to 0^+} f^{-1}(x) = \frac{-b}{a} = \pm 1$

متوسط و ابتکاری

گزینه ۳

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(۱۶)

n زوج $\longrightarrow n^+ \Rightarrow |x-(-n-1)| = |2n+1| = 2n+1$

$n^- \Rightarrow x-(n-1)+k = k+1$ — $k = 2n$

$(-n)^+ \Rightarrow |x-(n-1)| = |-2n+1| = 2n-1$

$(-n)^- \Rightarrow x-(-n-1)+k = k+1$

$k+1 = 2n-1$

زوج می‌تواند k باشد

k = فرد می‌تواند باشد

متوسط و ابتکاری

گزینه ۲

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(۱۷) متوسط

$\lim_{x \to 0} g(x) \frac{4x-1}{x} \Rightarrow \lim_{x \to 0} g(x) = f'(0)$

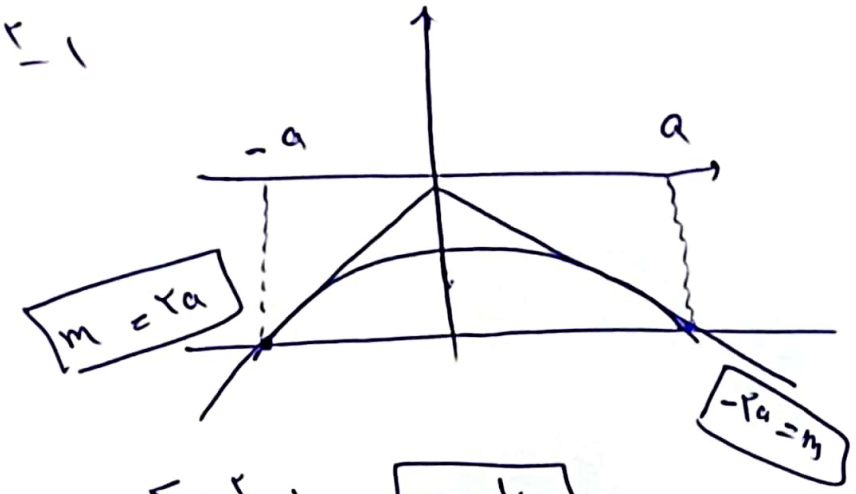
$f(x) = \left(\frac{-1+x}{1+x}\right)^2 = 2\left(\frac{2c}{(1+x)^2}\right)\left(\frac{-1+x}{1+x}\right) = (-4)$

گزینه ۳

۱۹

گزینه ۱

$y = -x^2 - 1$



$-۴a^2 - 1 \Rightarrow a = \frac{1}{2}$

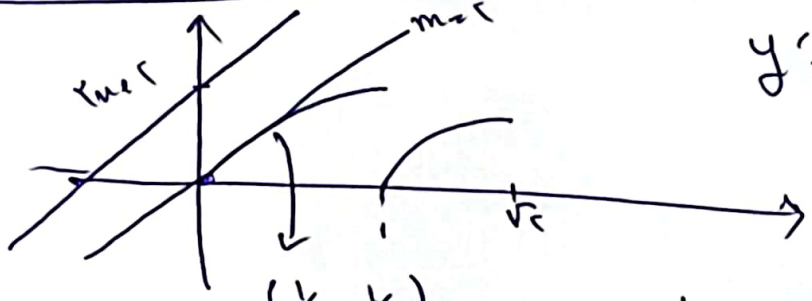
$y = -\frac{1}{2}(0)$

۲۰

$y' = \frac{1}{2\sqrt{x}} = 2$

$\sqrt{x} = \frac{1}{4}$

$x = \frac{1}{16}$



$d = \frac{\frac{1}{4} - \frac{1}{16} - 2}{\sqrt{5}} = \frac{\frac{19}{16}}{\sqrt{5}} = \frac{19\sqrt{5}}{80}$

گزینه ۲